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Incidental brane defects

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Abstract: In the models of brane construction, the isometry of a compactified space might be broken by branes. In four-dimensional effective lagrangian, the breaking of the isometry is seen as the spontaneous breaking of the corresponding effective symmetry. Then it seems natural to expect that there are various kinds of defects that will be implemented by the spontaneous symmetry breaking. These defects are parametrized by the brane positions. In this paper we consider two kinds of such "brane defects", which are formed by the local fluctuations of the locations of branes along their transversal directions. The fluctuation of a brane position might leads to winding (or wraping) around a non-contractible circle of the compactified space. These "primary" brane defects are already discussed by several authors. On the other hand, if there are multiple branes in the compactified space and their configuration in a compactified space is determined by the potential that depends only on their relative positions, one might find incidental symmetry in the effective potential, which is spontaneously broken by branes. We examined the latter "incidental" symmetry breakings and stable defect configurations. We paid special attention to the difference between "primary" brane defects.

Keywords: Solitons Monopoles and Instantons, Extra Large Dimensions, Cosmology of Theories beyond the SM, Cosmological Phase Transitions.

Contents

1. Introduction

In spite of the great success of quantum field theory and classical Einstein gravity, there is still no consistent unification scenario in which quantum gravity is successfully included. Perhaps the most promising scenario in this direction is string theory, in which consistency of the quantum gravity is ensured by a requirement of additional dimensions. Originally the size of extra dimensions was assumed to be as small as M_p^{-1}. However, later observations showed that there is no reason to require such a tiny compactification radius [1]. In this respect, what we had seen in the old string theory was a tiny part of the whole story. In the new scenario, the compactification radius (or the fundamental scale) is an unknown parameter that should be determined by observations. In models with large extra dimensions, the observed Planck mass is obtained by the relation $M_p^2 = M_*^{n+2}V_n$, where M_* and V_n denote the fundamental scale of gravity and the volume of the n-dimensional compact space. In this scenario the standard model fields are expected to be localized on a wall-like structure and the graviton propagates in the bulk. The most natural embedding of this picture in the string theory context is realized by a brane construction. Thus it is quite important to construct the models of the brane world where the observed spectrum of the standard model is included in the low energy effective theory. Moreover, we know that sometimes the cosmology of the models for the braneworld (or models with large extra dimensions) seems quite peculiar.[1] We know historically that the characteristic features of phenomonological models are revealed by discussing their cosmological evolutions.

In conventional models with more than four dimensions, compactified space has a topology K. The isometry of K is seen as a gauge symmetries of the effective four-dimensional theory. However, in theories where our world is a domain wall (or brane) embedded in the

[1] Constructing successful models for inflation with a low fundamental scale is still an interesting problem [2, 3]. Baryogenesis and inflation in models with a low fundamental scale are discussed in [4]–[6]. We think constructing models of particle cosmology with large extra dimensions is very important since we are expecting that future cosmological observations would determine the fundamental scale of the underlying theory.

extra space, one might expect that the isometry is broken by the existence of such object.[2] Even if an isometry of the compactified space remains as an exact symmetry in the effective lagrangian, it might be broken spontaneously by the existence of a brane. A brane localized in K breaks spontaneously the isometries of K, which is seen as a Higgs effect in the four-dimensional effective theory [7]. As a brane localized in K breaks the symmetry of the effective four-dimensional theory, one might expect that there exists a topologically nontrivial mapping of the brane position, which corresponds to strings or monopoles in conventional field theory. Of course, topological defects of similar mappings can be constructed within classical Kaluza-Klein setup without branes, such as Kaluza-Klein strings or monopoles. However, our main concern in this paper is to examine defects that are defined by a nontrivial mapping of the brane positions in extra dimensions. In this paper, we call them "brane defects".

In generic cases, brane defect is constructed from a non-contractible winding (or wrapping) of a brane position around extra dimension(s). In this case, the parameter of the mapping is defined by absolute coordinate in the compactified space. In this paper, we call them "primary" brane defects. Primary brane defects are discussed by several authors. Strings (including Alice strings) and monopoles are discussed in ref. [8], and skyrmions are discussed in ref. [9]. At first sight, one might feel some doubt about these configurations. Recalling a typical string configuration in conventional field theory, one will find that the broken symmetry is restored in the core. On the other hand, in brane defects that utilize the brane position in compactified space, one cannot simply expect such restoration in the core. Is there any possible mappings that might not be singular in the core? An answer to this question is given by Dvali et al. [8]. The authors constructed branes from the kink configurations of a scalar field and showed that the kink (brane) is delocalized in the core, which solves the naive singularity. These "primary" defects have many interesting properties. However, in generic models, a secondary weak inflation is sometimes required, which dilutes existing particles and defects. After the dilution of cosmological defects, it is hard for the defects to influence the later cosmology. In this case, some peculiar mechanisms will be required for the defects to be produced after secondary weak inflation. Thus, we think it is quite interesting if one could find brane defects that have the same (or similar) peculiar properties but are safely produced after weak inflation.[3]

In this paper, we focus our attention to the *relative* coordinates of branes. Our idea is quite simple. In general, relative coordinates might have *incidental* symmetries, which could be used to construct brane defects. When branes change their relative positions along extra dimensions as a consequence of local fluctuations, the effective symmetry of the relative brane-brane coordinate might allow stable defect formation in four-dimensional spacetime. We will show some explicit examples of these incidental brane defects. We then show that they are produced after thermal brane inflation [13]. For the simplest

[2]If the compactified space is warped by branes, effective symmetry is explicitly broken in low-energy lagrangian. In this case the masses of the pseudo-Nambu-Goldstone (pNG) bosons are determined by the warp factor [7].

[3]In this case one should be careful about cosmological constraints, because cosmological defects sometimes put serious constraint on the model, if they are produced [11, 12].

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example, let us consider two branes located in large S^1 that is perpendicular to the branes. Here we assume that the distance between the two branes is fixed by the mechanism that is discussed in ref. [13]. In this case, it is quite easy to see that degenerated vacua are formed when two branes exchange their positions along the extra dimension. If the exchange happens in a local domain of four-dimensional spacetime, a domain wall will appear on the surface of the domain, which interpolates between two degenerated vacua. This naive idea seems successful, which will be discussed in section 2 by using an explicit kink(brane) configuration of a scalar field. Now let us imagine that there are two large extra dimensions that are perpendicular to the branes. If the two branes wind around each other in the compactified space, a peculiar type of string will be formed in the four-dimensional spacetime. Along this line of thought, it is possible to construct monopole-like configurations when there are *three* extra dimensions. More peculiar example would be a time-dependent configuration that will correspond to the so-called Q-balls in conventional four-dimensional theory.

In general, the cosmological production of incidental brane defect will be easier than primary brane defects.[4] Some peculiar cosmological implications of these incidental brane defects are discussed in the forthcoming papers [15].

2. Domain walls

In this section we consider two types of domain wall configuration, which are produced by the variations of the relative brane positions. In the first example we consider two branes constructed from an explicit kink configuration of a scalar field, while in the second example we consider three *intersecting* branes.

Our first example[5] is a domain wall in the four-dimensional spacetime, which is produced by exchanging the positions of two domain walls (branes) along extra dimension. The two walls are located at the fixed points of an orbifold [16]. Here we consider the simplest model where a single real scalar field ϕ lives in five dimensions. The extra dimension x_5 is in the interval $[0, 2L]$. Imposing a boundary condition on the field that are periodic up to the $\mathbb{Z}_2$ symmetry of the lagrangian, the extra dimension becomes an orbifold. The

[4] In ref. [14], brane inflation with brane-brane interactions at an angle is analyzed. When branes collide, tachyon condensation appears and it generically allows the formation of lower-dimensional branes. After brane inflation, when the ground state string mode becomes tachyonic, the typical particle horizon size is larger than the sizes of the compactified dimensions, which means that the universe is homogeneous in the compactified dimensions. Thus, one might think that the Kibble mechanism does not produce defects in the compactified directions [14]. The resulting lower-dimensional defects are $D(p-2)$-branes wrapping the same compactified cycles as the original p-branes, with $(3-2)$ uncompactified dimension, which turn out to be the cosmological strings. Thus, they have concluded that other defects, such as domain walls or monopoles, are not produced after brane inflation. These statements are normally true, but there are some exceptions. The Kibble mechanism might take place in the uncompactified directions for the fluctuations of brane positions along extra dimensions. The productions of certain types of domain walls, monopoles and other defects are possible just after (thermal) brane inflation.

[5] See figure 1

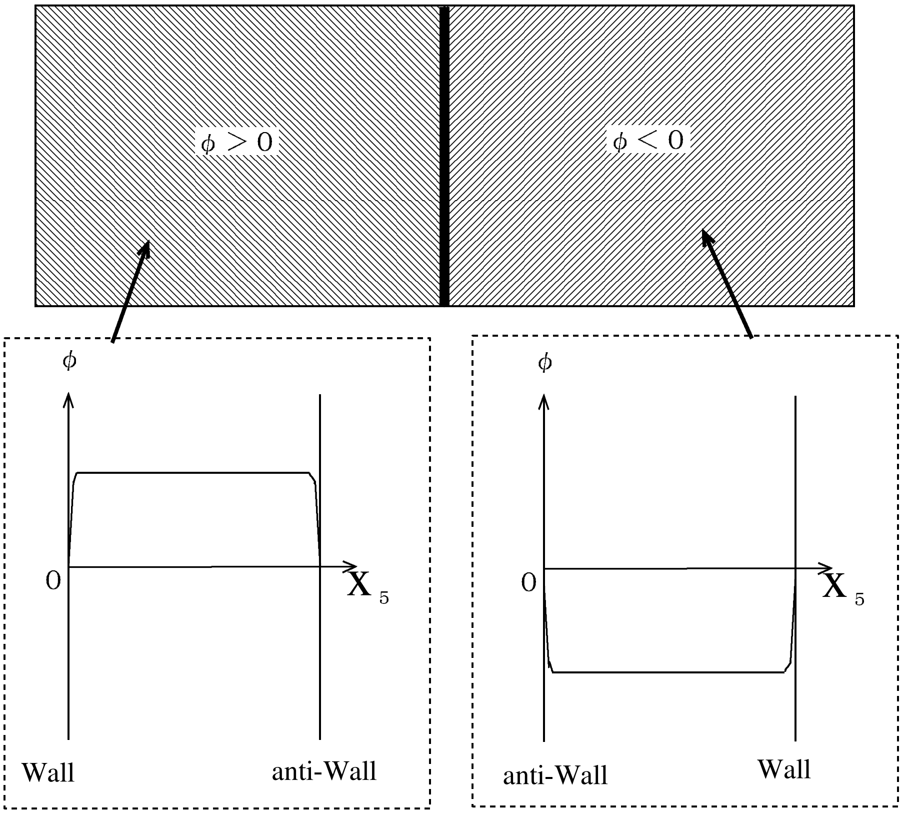

Figure 1: Domain wall that interpolates between two degenerated vacua, $\phi > 0$ and $\phi < 0$, is shown. Wall(brane) and anti-wall(anti-brane) located at each fixed point in the fifth dimension are exchanged when one passes through the center line in the upper picture.

lagrangian is

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2 , \tag{2.1}$$

where the coupling λ is real and $v^2 > 0$. The lagrangian is invariant under the transformation

$$\phi(x, x_5) \to \phi'(x, x_5) \equiv -\phi(x, L - x_5) . \tag{2.2}$$

The required boundary condition is

$$\begin{aligned} \phi(x, -x_5) &= \phi'(x, L - x_5) = -\phi(x, x_5) \\ \phi(x, L + x_5) &= \phi'(x, x_5) = -\phi(x, L - x_5) . \end{aligned} \tag{2.3}$$

Although the boundary condition (2.3) requires vanishing scalar field on the fixed points at $x_5 = 0$ and $x_5 = L$, the scalar field will develop a vacuum expectation value in the bulk. In this case, a kink (anti-kink) configuration appears at the fixed points. It is easy to find two degenerated vacua that is produced by flipping the sign of $\langle\phi\rangle$. When the sign of $\langle\phi\rangle$ is flipped along a direction of four-dimensional spacetime, the positions of the kink and the anti-kink in the fifth dimension will be exchanged. A domain wall is formed in four-dimensional spacetime, which interpolates between the domains of $\phi < 0$ and $\phi > 0$. In this case the degenerated vacua are formed by exchanging the positions of kink and anti-kink in the fifth dimension, which is induced by flipping the sign of $\langle\phi\rangle$ in the four-dimensional spacetime.[6]

Now let us consider what happens in the core of the incidental brane defect. In the core of the defect, the vacuum expectation value of the scalar field $\langle\phi\rangle$ vanishes. In this case, it will be natural to think that the walls(branes) on which we are living are delocalized in the core. As is discussed in ref. [8] for primary brane defects, the localized matter fields feel fifth dimension in the core. Thus, it acts as "holes" or "windows" to the extra dimensions.[7]

Even if we do not invoke an explicit construction of branes as defects from scalar fields, incidental domain walls might appear in any models where multiple branes live in one perpendicular dimension. Let us assume that in a local domain of four-dimensional spacetime branes exchange their positions along fifth dimension. If the new vacuum is (at least locally) stable, it is possible to construct domain walls interpolating between two domains in the four-dimensional spacetime. In this case, the exchanged branes will overlap in the core of the domain wall.[8] The "overlap" is interesting from a cosmological point of view, since the suppressed couplings might be enhanced in the core [4, 6, 18].

Let us discuss our second example. Another type of the incidentally degenerated vacuum might appear in the models of interesting brane world, where three branes are intersecting in the compactified dimensions. Here we consider the simplest example in ref. [19, 20]. What we would like to see is the Yukawa couplings in the quark sector. The Yukawa coupling among two chiral fermions and one Higgs boson cannot appear from the perturbative effects of the string theory, but induced by worldsheet instanton corrections for the corresponding triangle that has three boundaries of the intersecting branes and three vertices where matter fields live.

To be more concrete, here we consider the simplest case and derive the expression for Yukawa couplings. When computing a sum of worldsheet instantons, the simplest example comes from D-branes wrapping 1-cycles in a T^2, where branes are intersecting at one angle.

[6]To make domain walls cosmologically harmless, the energy difference ϵ between two quasi-degenerated vacua must satisfy the condition, [11]

$$\epsilon \geq \frac{\sigma^2}{M_p^2}, \tag{2.4}$$

where σ is the tension of the domain wall. The degeneracy is broken if there is another scalar field ϕ_2 that satisfies the same boundary condition. Although the $\mathbb{Z}_2^{\rm sim}$ symmetry that corresponds to the *simultaneous* flips of $\langle\phi\rangle$ and $\langle\phi_2\rangle$ will remain, the $\mathbb{Z}_2$ symmetries of their *independent* flips are explicitly broken if there is an effective cross term $\sim (\phi\phi_2)^{2n+1}$, where n is integer. In this case, the role of ϕ_2 corresponds to a constant that is called "odd mass" in ref. [17].

[7]This point is discussed in [8] for primary brane defects. However, domain walls are not discussed in previous papers.

[8]In the explicit brane construction from scalar fields, there are at least two types of symmetry breaking. As a brane is constructed by a kink configuration of a scalar field, a symmetry breaking occurs when the scalar field develops non-zero vacuum expectation value. Branes are produced at this time. If supersymmetry remains in the bulk or at least softly broken, the interaction between two branes might be well suppressed. In this case one can expect that thermal effects might stabilize two or more branes to coincide at a point [13]. Then the second symmetry breaking occurs when branes fall apart. Assuming supersymmetry, the scales of the two phase transitions will be hierarchically discriminated.

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Here we associate each brane to complex number $z_\alpha, (\alpha = a, b, c)$,

$$\begin{aligned} z_a &= R \times (n_a + \tau m_a) \times x_a \\ z_b &= R \times (n_b + \tau m_b) \times x_b \\ z_c &= R \times (n_c + \tau m_c) \times x_c \,. \end{aligned} \tag{2.5}$$

Here $(n_\alpha, m_\alpha) \in \mathbb{Z}^2$ denote the 1-cycle the brane α wraps on T^2 and $x_\alpha \in \mathbb{R}$ is an arbitrary number. τ is the complex structure of the torus. These branes are given by a straight line in C. The triangle corresponding to a Yukawa coupling must involve three branes, which has the form (z_a, z_b, z_c) with $z_z + z_b + z_c = 0$. The solution is

$$x_\alpha = \frac{I_{\beta\gamma} x}{d}\,, \tag{2.6}$$

where $x = x_0 + l, x_0 \in \mathbb{R}$, $l \in \mathbb{Z}$ and $d = g.c.d.(I_{ab}, I_{bc}, I_{ca})$. Here $I_{\beta\gamma}$ stands for the intersection number of branes β and γ. Indexing the intersection points, one can obtain a simple expression for x_0,[9]

$$x_0(i,j,k) = \frac{i}{I_{ab}} + \frac{j}{I_{ca}} + \frac{k}{I_{bc}} + \frac{I_{ab}\epsilon_c + I_{ca}\epsilon_b + I_{bc}\epsilon_a}{I_{ab} I_{bc} I_{ca}}\,, \tag{2.7}$$

where the parameter ϵ_α correspond to shifting the positions of the three branes. Using this solution, one can compute the areas of the triangles whose vertices lie on the triplet of intersections (i, j, k),

$$A_{ijk}(l) = \frac{1}{2}(2\pi)^2 A |I_{ab} I_{bc} I_{ca}| \left(x_0(i,j,k) + l\right)^2\,, \tag{2.8}$$

where A represents the Kähler structure of the torus. The corresponding Yukawa coupling is given by

$$Y_{ijk} \sim \sigma_{abc} \sum_{l \in Z} \exp\left(-\frac{A_{ijk}(l)}{2\pi\alpha'}\right)\,, \tag{2.9}$$

where $\sigma_{ijk} = \mathrm{sign}(I_{ab} I_{bc} I_{ca})$ is a real phase.

Now our question is how one can determine the areas of the triangles. A perturbative force between branes can produce potential for the distance between two branes. However, it is obvious that this force cannot affect the area of a triangle when branes are *intersecting*. On the other hand, one can see from eq. (2.9) that almost all the parameters are determined if the windings of the branes and the structure of the manifold are fixed by some mechanisms. The only ambiguity that might remain at low energy effective theory is one parameter of three ϵ_α, which corresponds to shifting the relative brane position. For the area of a triangle, only one of the three parameters ϵ_α is independent.

An effective potential for the area of a triangle is obtained by considering a well-known 1-loop correction from fermion loops, [21]

$$\Delta V(\phi_c) = -\frac{3}{64\pi^2} Y_{ijk}^4 \phi_c^4 ln\left(\frac{\phi_c^2}{\mu^2}\right)\,, \tag{2.10}$$

where ϕ_c denotes the classical field. From eq. (2.10) and (2.9), one can easily see that the 1-loop correction stabilizes the area of the triangle.[10]

[9] See ref. [20] for more detail.

[10] See figure 2

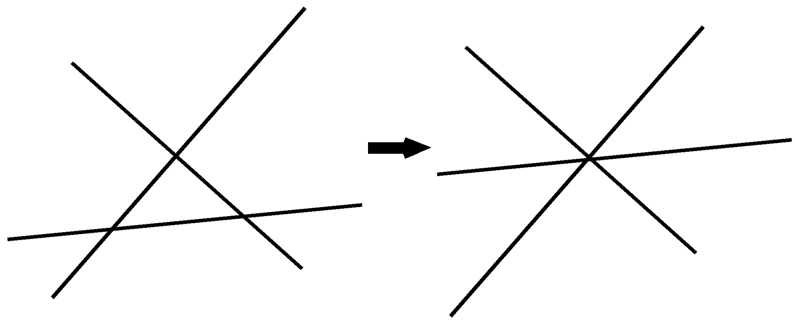

Figure 2: The area of a triangle in the left picture determines the Yukawa coupling of the fields that live on each vertex. Considering 1-loop correction, the triangle tends to shrink to a point, which is shown in the right picture.

Because of the exponential form of the potential, intersecting branes will be stabilized when one of the areas of the three triangles vanishes. Because of phenomonological requirements and the corresponding brane setups, three triangles cannot shrink simultaneously to a point. Thus it is possible to construct stabilized models in which one of the three Yukawa couplings becomes large, while others remain (hierarchically) small. In general, at least three triangles are included in models for intersecting braneworld, which correspond to three generations in the standard model. Then it leads to (at least) three degenerated vacua where each triangle might shrink. If three generations are geometrically equal, there is no sensible reason why the third generation is cosmologically selected to become the heaviest. These degenerated vacua suggest the existence of incidental domain walls.

Here we stress that our idea in this section is quite generic. When there are multiple branes in the compactified space, their positions must be determined by some mechanisms. The most obvious forces are induced by interactions between two distant branes, which produce an effective potential that depends only on the absolute value of the distances. If there are many branes in the compactified space and their configuration is determined by the potential of this type, it is quite natural to expect incidental domain walls that are produced by the fluctuations of the brane positions along extra dimensions. The situation is similar to the conventional defect in the crystal. In general, as we have discussed above, interactions among more than two branes are less effective than the perturbative force between two branes. As an example, we considered triangle interaction among intersecting three branes and showed that it is possible to construct domain walls in intersecting brane models.

3. Strings

The primary brane defects that were discussed in [8, 9] are constructed by a single brane position along extra dimensions. On the other hand, in more generic situations, one would expect multiple branes in a compactified space. Thus our starting point in this section is to add branes to the well-known models and examine the difference. Here we mainly consider a compactified space S^2. If there is only one brane in the compactified

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space, the symmetry breaking is SU(2) $\to$ U(1), which forbids stable string configuration. Let us recall that in conventional field theory many models have been discussed to circumvent this difficulty. For example, if the symmetry is broken to SU(2) $\to$ U(1) $\times$ $\mathbb{Z}_2$, one can find stable string configuration [22]. This possibility is examined in ref. [8] by adding an additional brane at the opposite pole of S^2. Identifying two branes, one can obtain $\mathbb{Z}_2$ symmetry.

First we consider a looser condition. For simplicity, we assume large S^2 where two branes are located. Distance between branes is assumed to be stabilized by a mechanism. We also assume that the distance between the two branes is stabilized at a distance scale smaller than the radius of the compactified space.[11] Considering the idea of thermal brane inflation [13], it is natural to expect that the two branes might glue together to a point by thermal effects during a period of the Universe. During this period, the symmetry is restored to U(1). Then the two branes will fall apart, which induces symmetry breaking U(1) $\to I$, because we have assumed that the distance between branes is smaller than the radius of the compactified space. If one considers a limit where one brane is much heavier than the other, one can easily find an effective description of the "light" brane fluctuation around a probe brane, which becomes quite similar to the conventional field theory in the lowest order expansion [7, 23]. In this case, an incidental string is formed by the position of a light brane, parametrized by the windings around the heavy brane. Unlike the primary strings that winds around S^1 compactified space, one do not have to worry about singularity at the string core. In the core of the incidental brane string, two branes will coincide to restore the U(1) symmetry. This point is quite different from the primary string that requires delocalization of the brane in the core to solve the singularity.

To make our discussion more clear, here we consider an explicit example. As we are considering "brane" that might be delocalized in the core, it will be helpful to consider vortices from a scalar field in S^2 compactified space. An explicit construction of such vortices is already discussed in [24]. Here we do not repeat the details of the calculations. We simply examine the obtained solution for three vortices. The model is defined in the spacetime $\mathbb{R}^n \times S^2$, where S^2 is a two-dimensional sphere of radius r. The spherical coordinates of S^2 are denoted by (θ, ϕ). The model is consisted of a complex scalar field f in $\mathbb{R}\times S^2$ accompanied by a gackground gauge field A in S^2, which represents the Dirac-Wu-Yang monopole [25]. The vacuum configuration of the scalar field minimizes the classical energy functional

$$E = \int d\theta d\phi r^2 \sin\theta \left[\frac{1}{r^2}\left|\frac{\partial f_\pm}{\partial \phi}\right|^2 + \frac{1}{r^2 \sin^2\theta}\left|\frac{\partial f_\pm}{\partial \phi} - iq(\pm 1 - \cos\theta) f_\pm\right|^2 - \mu^2 f_\pm^* f_\pm + \lambda (f_\pm^* f_\pm)^2\right]. \tag{3.1}$$

In this section we examine the case of $q = 3/2$, where three vortices appear in S^2.[12] The

[11] A possible mechanism for stabilization is discussed in ref. [13]. If the distance is not stabilized but there is only a repulsive force between them, two branes will repel to north and south poles. In this case, unwanted U(1) will remain.

[12] See ref. [24, 25] for more details.

lowest expansion in a series of the eigenfunction is

$$f^{3/2}_{\pm}(\theta,\phi) = \left[c_{3/2}e^{-3i\phi/2}\cos^3\left(\frac{\theta}{2}\right) + c_{1/2}3^{1/2}e^{-i\phi/2}\cos^2\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) + \right.$$
$$\left. + c_{-1/2}3^{1/2}e^{i\phi/2}\cos\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right) + c_{-3/2}e^{3i\phi/2}\sin^3\left(\frac{\theta}{2}\right)\right]e^{\pm 3i\phi/2}. \quad (3.2)$$

From eq. (3.1) and eq. (3.2), one can obtain the energy functional. The minimum is found at

$$(c_{3/2}, c_{1/2}, c_{-1/2}, c_{-3/2}) = (-v, 0, 0, v). \quad (3.3)$$

From eq. (3.3) and eq. (3.2), one can find the location of the zero points of the scalar field f are located at $\phi = 0,\ 2\pi/3,\ \pi/3$ on $\theta = \pi/2$, which are determined up to reparametrization invariance.

Before arguing about string defect configuration, we should consider cosmological evolution that makes string production possible. Although cosmological mechanism for producing brane defects is still unclear, we know at least one possible mechanism. Assuming that the idea of thermal stabilization in ref. [13] works in our case, three branes will glue together to a point during a period of the Universe. Below the critical temperature, the three branes begin to fall apart and the symmetry is spontaneously broken.[13]. The incidental string formed in this way will be related to the windings of ϕ.

Let us consider a straight string configuration along z-axis in four-dimensional spacetime. Here we introduce polar coordinates r_s and ϕ_s in four-dimensional spacetime. We introduce a new variable α and rewrite the configuration (3.3) as

$$(c_{3/2}, c_{1/2}, c_{-1/2}, c_{-3/2}) = \left(-\sqrt{2}v\sin\frac{\alpha}{2}, 0, 0, \sqrt{2}v\cos\frac{\alpha}{2}\right). \quad (3.4)$$

It is easy to see that the three vertices are settled at their vacuum when $\alpha = \pi/2$. For $\alpha = 0$, three vertices coincide at $\theta = 0$, which corresponds to the core of the string.[14] It is straightforward to find explicit string configuration by using $\alpha(r_s)$. In our example, the core structure of the incidental string configuration is different from the primary brane defects, where "brane" must be delocalized in the core. If the $\mathbb{Z}_3$ symmetry of the three vortices is an exact symmetry, one will find alice string by identifying $\phi = \phi_s/3$. On the other hand, however, one should be careful about the original motivation for constructing models with three vortices on S^2. These models are interesting because it might reproduce the required three generations of the standard model. Because we know that the three generations in the standard model are not equal, we think it should be natural to assume that the effective $\mathbb{Z}_3$ symmetry is explicitly (but softly) broken in more phenomenological

[13]We should be careful about the above assumption. In ref. [13], supersymmetry is assumed in the bulk so that the forces between branes are hierarchically suppressed. In this case the time (t_1) when vortices(branes) are produced is much earlier than the time (t_2) when vortices begin to fall apart, which justifies the analysis in ref. [13]. On the other hand, however, as we are not considering explicit supersymmetry in our toy model, the condition $t_2 \gg t_1$ is not clear. Thus in the above example, where supersymmetry is not included explicitly, we have to assume that the formation of the three vortices occurs at least simultaneously at a point in S^2.

[14]Schematic pictures are given in figure 3.

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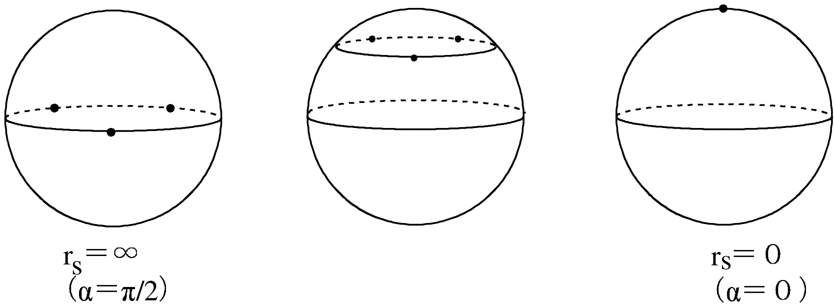

Figure 3: Vortices are denoted by blobs. In the left picture, which corresponds to $r_s \to \infty$, vortices are placed at the equator of S^2, $\theta = \pi/2$. In the right picture, which corresponds to $r_s \to 0$, all vortices are placed at the north pole, $\theta = 0$.

settings. Even if the $\mathbb{Z}_3$ symmetry is explicitly broken and the regular triangle is warped in the low-energy effective theory, one can construct string configuration by identifying $\phi = \phi_s$.

Finally we will discuss why these incidental brane defects are visible for an observer on a brane. As the defects that we have discussed above are formed by the parameters *in the bulk*, the observation of such defects is not a trivial issue. Of course these defects are invisible if the physics on our brane is irrespective of the parameters that are shifted in the defect. To see such defects, at least one of the parameters are required to be detectable on our brane. For example, in the above example for a string, one might assume some fields in the standard model are localized on each branes and their interactions are suppressed by the distance between branes. In this case, since the three branes coincide in the core of the defect, observable enhancement of the interaction will appear in the scatterings mediated by the string.

4. Conclusions and discussions

In the models of brane construction, the isometry of a compactified space might be broken by branes. In four-dimensional effective lagrangian, the breaking of an isometry is seen as the spontaneous breaking of the corresponding effective symmetry. Then one might naturally expect that there are various kinds of defects that will be implemented by the spontaneous symmetry breaking. These defects are parametrized by brane positions, and will be classified into two categories. Primary brane defect is formed by a local fluctuation of the position of a brane, which winds (or wraps) around a non-contractible circle of a compactified space. The structure of a primary brane defect is solely determined by the isometry of the compactified space. On the other hand, incidental brane defect is formed by the local fluctuations of the relative brane positions among more than one brane. We showed some explicit examples of incidental brane defects and discussed differences between primary defects.

In this paper we did not consider time-dependent configurations, such as Q-balls or instantons. We think it is quite interesting to discuss the cosmological consequences of "incidental" brane Q-balls that might be produced after chaotic brane inflation.

Acknowledgments

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